**Binomial Distribution**:

The binomial distribution is a discrete probability distribution that models the number of successes in a fixed number of independent and identical trials, where each trial has only two possible outcomes: success or failure.

The probability mass function (PMF) of a binomial random variable X, which represents the number of successes in n trials, with the probability of success in each trial being p, is given by:

P(X = x) = (n choose x) \* p^x \* (1-p)^(n-x), for x = 0, 1, 2, ..., n

where:

* n is the number of trials
* x is the number of successes
* p is the probability of success in a single trial
* (1-p) is the probability of failure in a single trial

Mean (Expected Value) of the Binomial Distribution: The mean or expected value of a binomial random variable X is given by:

E(X) = n \* p

This means that the expected number of successes in n trials is simply the product of the number of trials (n) and the probability of success in a single trial (p).

**Example**:

Suppose a fair coin is tossed 10 times. Let X be the number of heads obtained. Since the coin is fair, the probability of getting a head on any single toss is 0.5.

Here, n = 10 (number of trials), and p = 0.5 (probability of success, which is getting a head).

The mean or expected value of X is: E(X) = n \* p = 10 \* 0.5 = 5

This means that the expected number of heads in 10 tosses of a fair coin is 5.

**Poisson Distribution**:

The Poisson distribution is a discrete probability distribution that models the number of occurrences of an event in a fixed interval of time or space, given that the events occur independently and at a constant average rate.

The probability mass function (PMF) of a Poisson random variable X, which represents the number of occurrences of an event in a fixed interval, with an average rate of λ occurrences per interval, is given by:

P(X = x) = (e^(-λ) \* λ^x) / x!, for x = 0, 1, 2, ...

where:

* x is the number of occurrences
* λ (lambda) is the average rate of occurrences per interval
* e is the base of the natural logarithm (approximately 2.71828)

Mean (Expected Value) of the Poisson Distribution: The mean or expected value of a Poisson random variable X is equal to the average rate of occurrences, λ.

E(X) = λ

**Example:**

Suppose the average number of defective items produced in a manufacturing process is 2 per hour. Let X be the number of defective items produced in a given hour.

Here, λ = 2 (the average rate of occurrences per interval, which is 1 hour).

The mean or expected value of X is: E(X) = λ = 2

This means that the expected number of defective items produced in a given hour is 2.

**Mean Property:**

One important property of the mean or expected value is that it is a linear operator. This means that for any two random variables X and Y, and any constants a and b, we have:

E(aX + bY) = a \* E(X) + b \* E(Y)

This property is useful in various calculations and derivations involving random variables and their expected values.

Example: Let X and Y be two independent random variables, with E(X) = 3 and E(Y) = 2. Consider the random variable Z = 2X - 3Y.

Using the mean property, we can find the expected value of Z as:

E(Z) = E(2X - 3Y) = 2 \* E(X) - 3 \* E(Y) = 2 \* 3 - 3 \* 2 = 6 - 6 = 0

This example demonstrates how the mean property can be used to calculate the expected value of a linear combination of random variables.

**In summary**,

the binomial and Poisson distributions are essential discrete probability distributions, with their means or expected values being simple functions of their respective parameters. The mean property provides a convenient way to calculate the expected values of linear combinations of random variables, making it a valuable tool in probability theory and its applications.